

# Modeling and Characterization of Supercapacitor in Many Power Electronics Application

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**Abstract**— As a new technology supercapacitors are emerging device for power electronics application. It acts as an energy buffer or energy sources due to their high power density compared to other charge storing devices. In this paper an equivalent circuit model (ECM) is proposed by considering leakage and self-discharge current. Here all the electrical parameters of supercapacitor deduce in two phases .In first phase we didn't consider the leakage or self -discharge current and in later phase considered these losses. The proposed model is based on the supercapacitor RC branches model .Moreover, simulation results are presented to prove the validity of this model. This model can be used to determine or predict the actual energy obtained from a supercapacitor. This model also can be used in many industrial applications where supercapacitors act as an energy storage device.

**Index Terms**— Supercapacitor, Electric Double Layer Capacitor (EDLC), Equivalent Circuit Model (ECM), Leakage Loss, Self Discharge Loss, Kirchhoff's voltage law (KVL),Laplace Transform ,Inverse Laplace Transform

## 1 INTRODUCTION

In the contemporary world energy has become a primary focus of the major power and scientific community due to the changing global scenario. There has been great interest in developing and refining more efficient energy storage devices. Conventional batteries and capacitors are widely used for energy storing. Although conventional batteries have very high energy density but the lower cyclic lifetime and low power density restricts their application in high power system [1]. The instantaneous discharging behavior and low energy density reduce the reliability of the conventional capacitors in many applications. Compared with batteries and conventional capacitors, supercapacitors are more preferable due to their high-efficiency, lightweight, larger current density, wider working temperature range, less maintenance and environmental friendly [1]. To compare the performance of different storage devices Ragone plot is given below.

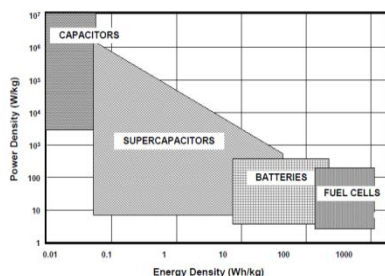


Fig.1. Comparison of storing devices using Ragone plot

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Considering all these advantages supercapacitors have become an attractive power solution for an increasing number of applications such as electric/hybrid electric vehicles, fuel cell vehicles, computer systems, UPS systems and power conditioners etc. Moreover, supercapacitors can be used for desalting water based on periodic sorption and desorption of ions on the extensive surface of porous electrode based on its capacitive properties [2], [3]. To reduce the time and costs for physical experimentation, the scientific community has exploited equivalent circuit modeling to predict characteristics of supercapacitors. In this paper an equivalent circuit model (ECM) is proposed to represent the characteristics of EDLC. This model is based on the supercapacitor RC branches model which is useful for most of the studies and applications [4], [5]. Here we developed all the electrical parameters of supercapacitor without and with considering leakage or self-discharge phenomenon respectively. This model was validated with supercapacitors simulation plots. In this paper we first introduce energy storing principle of supercapacitor and equivalent circuit models in section II. In section III we have discussed about proposed method and validity of this model is also presented in this section. Conclusion of this study is given in section IV.

## 2 ABOUT SUPERCAPACITOR

### 2.1 Energy Storage Principle

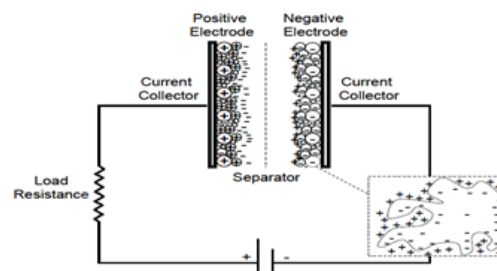


Fig.2. Charge storage principle of supercapacitor

Supercapacitors are constructed from two carbon based electrodes, an electrolyte, and a separator. Fig.2 represents the charge storage principle of supercapacitor. Like conventional capacitors, supercapacitors also store charge electro statically, and there is no transfer of charge between electrode and electrolyte. When voltage is applied, an electric field is created in the electrolyte which causes polarization in the electrolyte. Due to the natural attraction of unlike charges, ions in the electrolyte diffuse across the separator into the pores of the electrode of opposite charge. Thus, a double-layer of storing charge is produced at each electrode. As a result surface area is increased and decreases the distance between electrodes, which allow supercapacitor to achieve higher energy densities than conventional capacitors [9],[10],[11]. Electric energy was stored into the electrical double layer which was formed at the porous solid electrode/electrolyte interface [6]. Here separator acts as an insulator which prevents physical contact of electrodes but allows ion transfer between them [7]. Energy storing capacity of supercapacitor does not depend only on the specific surface area of electrode materials, but it also depends on the utilization rates of micro-hole of the porous electrode and the active materialism electrolyte [8]. Different types of carbon materials used as an electrode such as activated carbon, carbon aero gels, carbon nanotubes and metal oxides. As an electrolyte aqueous H<sub>2</sub>SO<sub>4</sub> or KOH is used.

## 2.2 Supercapacitor Equivalent Circuit Model

The purpose of equivalent circuit model is to represent the characteristics of supercapacitors in many power electronics applications. Three major considerations take into account before modeling equivalent circuit structure.

1. The model structure should be related to the physics of the device, but it should be as simple as possible to provide the engineer with a practical tool for design.
2. The model should describe the terminal behavior of the supercapacitor over the range of 30 min with sufficient accuracy.
3. It should be possible to determine the parameters of the proposed model using measurements at the supercapacitor terminals [4].

Equivalent circuit models are mainly classified into three categories. These are RC circuit model, supercapacitor RC branches model and a transmission line model. In this paper

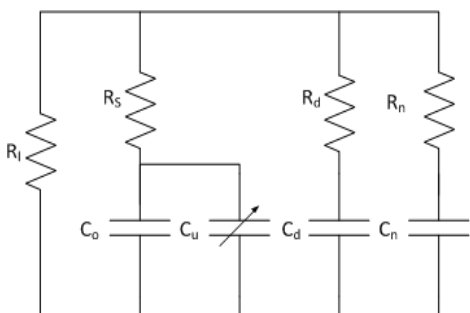


Fig.3: Supercapacitor RC branches model

Based on the physics of the device, supercapacitors modeled as an RC circuit. In fig.3 resistive element represents the resistivity of the electrode (carbon particles) materials of the supercapacitor and capacitive element represents the capacitance between carbon particles and electrolyte. Capacitance of the supercapacitor depends on potential differences across the material [3]. To represent this voltage dependence capacitance immediate branch is modeled as a voltage dependent differential capacitor. This capacitor is modeled as a constant capacitor  $C_o$ , with a parallel capacitor  $C_u$ , which is linearly dependent on the voltage  $V_c$  across it [12].

$$C_v = C_o + K_v |V_c| \quad (1)$$

Parameter  $K_v$  depends on numerous parameters, such as pore sizes, membrane porosity and packaging technology [4] and having the unit of Farad/V. This immediate branch represents the behavior of the supercapacitor in the time range of seconds in response to a charge action. The delayed branch with parameters  $R_d, C_d$  represent the behavior in the time range of minutes and the long term branch with parameters  $R_n, C_n$  represent the behavior in the time range more than 10 minutes. A leakage resistor  $R_l$  parallel to the terminal represents leakage loss or self-discharge loss of a supercapacitor.

## 3 PROPOSED MODELING AND VALIDITY OF THE MODEL

In this paper an equivalent circuit model is presented which is used to determine the actual behavior of supercapacitor in many power electronics application. During the dynamic charge and discharge process a small amount of current known as self-discharge current induced by leakage resistance  $R_l$ . So only the immediate branch and leakage resistance is considered in this paper.

### 3.1 Modeling of Charging Process

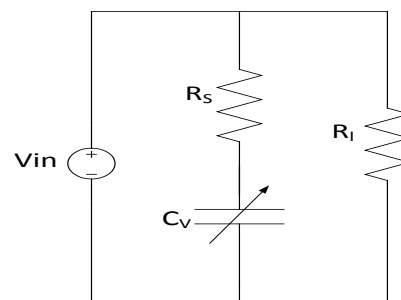


Fig.4. Modeling of charging process

We start the analysis by applying Kirchhoff's voltage law; the input voltage must be equal to the voltage drop over the resistor, plus the voltage across the capacitor. This can be expressed mathematically as,

$$V_{in} = V_{R_s}(t) + V_c(t) \quad (2)$$

Voltage across capacitor,

$$V_c(t) = \frac{1}{C_v} \int_0^t i(t) dt + V_c(0) \quad (3)$$

Therefore (2) becomes,

$$V_{in} = i(t) * R_s + \frac{1}{C_v} \int_0^t i(t) dt + V_c(0) \quad (4)$$

In this  $V_c(0)$  refers to the initial voltage of the capacitor and  $i(t)$  is the charging current. This current must be obtained before the capacitor voltage can be calculated. So by applying Laplace and Inverse Laplace Transform in (4) we get Charging current,

$$i(t) = \frac{[V_{in} - V_c(0)]}{R_s} * e^{\frac{-t}{R_s C}} \quad (5)$$

If  $V_c(0)$  negligible then,

$$i(t) = \frac{V_{in}}{R_s} * e^{\frac{-t}{R_s C}} \quad (6)$$

From (3) & (6) we get the charging voltage,

$$V_c(t) = [V_{in} - V_c(0)] e^{\frac{-t}{R_s C}} + V_c(0) \quad (7)$$

If  $V_c(0)$  is negligible (7) becomes

$$V_c(t) = V_{in} \left( 1 - e^{\frac{-t}{R_s C}} \right) \quad (8)$$

This represents the charging voltage without considering leakage loss. Leakage current flowing through the  $R_l$

$$i_l = \frac{V_{in}}{R_l}$$

So the actual charging current,

$$i_{ac} = i(t) - i_l \quad (9)$$

Equation (9) can be written as,

$$\left( \frac{V_{in}}{R_s} e^{\frac{-t}{R_s C}} - \frac{V_{in}}{R_l} \right) = C_v \frac{dV_c(t)}{dt} \quad (10)$$

By taking integral on both sides we get the actual charging voltage

$$V_{ac}(t) = V_{in} \left[ 1 - \frac{t}{R_l C} - e^{\frac{-t}{R_s C}} \right] \quad (11)$$

As energy is the integral of voltage and current, so the energy of a supercapacitor is

$$E = \int_0^t V(t) i(t) dt \quad (12)$$

From equation (6), (8) & (12) we get the charging energy without considering leakage

$$E_c = C_v V_{in}^2 \left[ -e^{\frac{-t}{R_s C}} + 0.5 + 0.5 e^{\frac{-t}{R_s C}} \right] \quad (13)$$

And from equation (9), (11), (12) & (13) we get the actual charging energy

$$E_{ac} = E_c - i_l \left[ t + R_s C_v \left( e^{\frac{-t}{R_s C}} - 1 \right) \right] \quad (14)$$

### 3.2 Modeling of Discharging Process

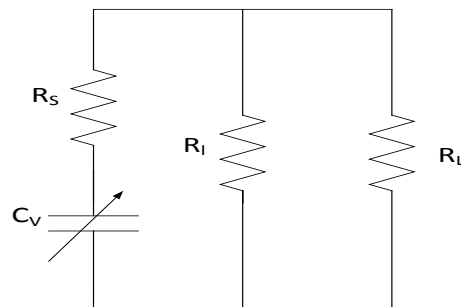


Fig.5. Modeling of discharging process

During discharging process, supercapacitor  $C_v$  acts as a power source and we consider it will discharge through a load  $R_L$ . Fig.5 can be simplified as,

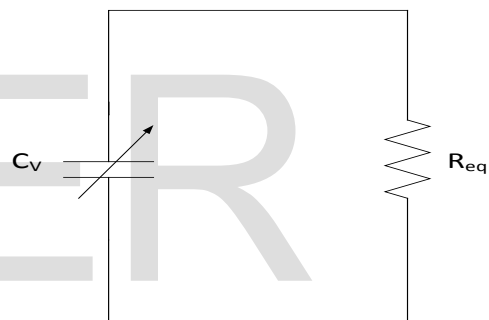


Fig.6. Simplified discharging circuit

Where,  $R_{eq} = R_s + (R_l \parallel R_L)$ . By applying KVL we can write in this circuit,

$$\frac{Q(t)}{C_v} - i(t) R_{eq} = 0 \quad (15)$$

Here  $i(t)$  is the total discharging current and  $Q(t)$  is the amount of charge stored during charging process. Again,

$$i(t) = \frac{dQ}{dt} \quad (16)$$

Plug (16) into (15), total discharging current is expressed as

$$i(t) = \frac{V_c(t)}{R_{eq}} e^{-t/R_{eq} C_v} \quad (17)$$

But a portion of this discharging current will flow through the leakage resistor which is appeared as a self-discharge loss. So the actual current flowing through the load is

$$i_L(t) = \frac{R_L}{R_L + R_L} i(t) \quad (18)$$

Predicted energy of the load is  $E_L = \int_0^t i^2(t) R_L dt$

$$E_L = \left( \frac{V_c(t)}{R_{eq}} \right)^2 \frac{R_{eq} C_v}{2} R_L \left[ 1 - e^{-\frac{2t}{R_{eq} C_v}} \right] \quad (19)$$

But a portion of this energy will be lost due to self-discharge, so the actual energy of the load is

$$E_L = \int_0^t i_L^2(t) R_L dt$$

$$E_L = \left( \frac{R_L V_c(t)}{(R_L + R_L) R_{eq}} \right)^2 \frac{R_{eq} C_v}{2} R_L \left[ 1 - e^{-\frac{2t}{R_{eq} C_v}} \right] \quad (20)$$

### 3.3 Validity of this Proposed Model

To prove the validity of proposed modeling we used BCAP350 D Cell supercapacitor

Table 1  
Parameters of BCAP350 D Cell

Rated Voltage	2.5 V
Capacitance	350 F
Internal Resistance	0.1 $\Omega$
Leakage Resistance	8.3 K $\Omega$

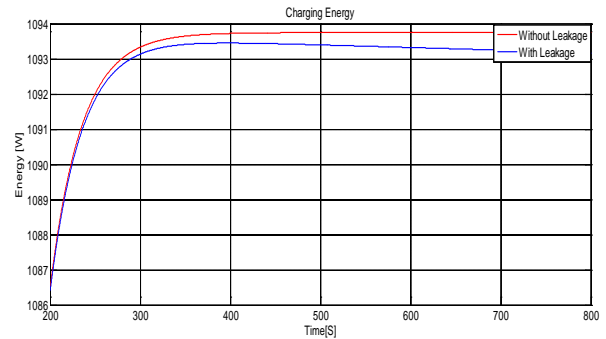
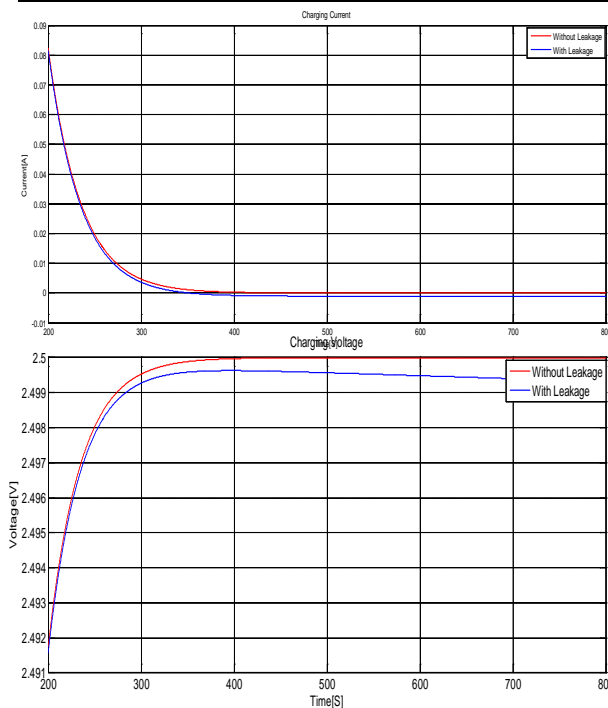


Fig.7. Validity of modeling during charging process

The proposed method was validated with supercapacitor charging and discharging plots. In all these plots we can see that two colors plots, one is red which represent measurement of parameters without considering leakage and another is blue color represent measurement with considering leakage. In this modeling we tried to show the actual condition of supercapacitor charging and discharging parameters by considering leakage or self-discharge property and we also differentiated these measurement with the parameters where these loss was not accounted. We can see that in Fig.7 charging voltage and energy curve after a certain point start to decrease. This is because at that point supercapacitor is fully charged. If we continue the supply after this point voltage or energy will be decreased due to unnecessary current flow through  $R_L$ . From charging current plot it is observed that after a fully charged point current will go negative region. This represents that after that point actual charging current is almost zero but an unnecessary current due to the extra charging causes falling of the original current.

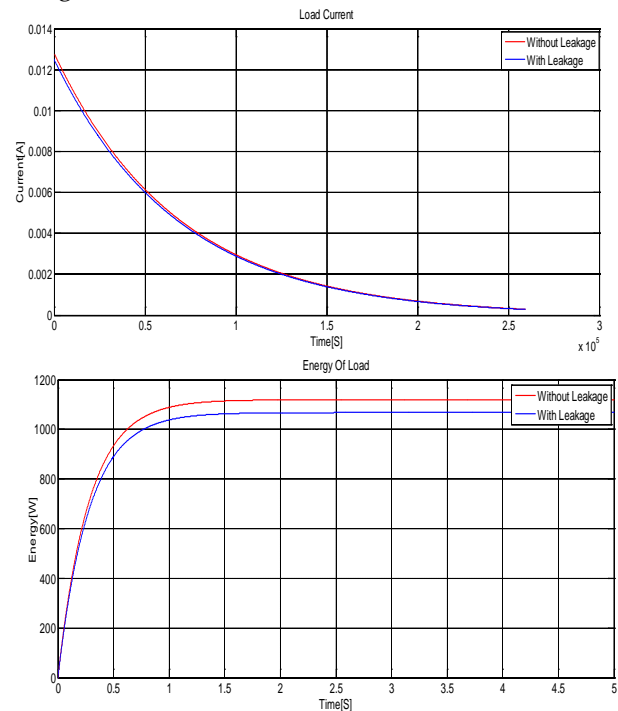


Fig.8. Validity of modeling during discharging process

During discharging supercapacitor acts as an internal power source, and it will discharge through a load. In this paper as a load we used a resistance of  $200\Omega$  which is represented by  $R_L$ . Top plot in Fig.8 we can see that load current is slightly went down denoted by blue color compared to red color current curve which should flow through the load. This is due to self-discharging current. Although it is desirable total energy of a charged supercapacitor will dissipate through the load but due to self-discharge loss a small amount of energy is lost and the actual energy curve represented by blue color is slightly went down compared to red color.

## 5 CONCLUSIONS

What is the basic difference between a capacitor and supercapacitor? Both can store energy while one discharges instantaneously the other tends to discharges in a way similar to that of a battery. But if it does so, it shall be better than a battery because it is much more environment friendly, less expensive and more over maintenance and hazard free. All these qualities have driven the scientists and engineers worldwide to modeling and developing the design of supercapacitors for application in power and electronic engineering. In reality supercapacitors are advanced version of conventional capacitors. So our effort in modeling and developing the supercapacitor is also participating with the world scientists and engineers in an effort to develop the model of a supercapacitor. In this paper an equivalent circuit model is proposed considering leakage or self-discharge loss. This model can be used to observe the actual behavior of supercapacitor in many power electronics application.

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## REFERENCES

- [1] Ran Niu and Hai Yang, "Modeling and Identification of Electric Double Layer Supercapacitors", ICRA Communication ( IEEE Conference ), pp. 1 -4 , 2011.
- [2] A.M. Johnson and J. Newman, "Desalting by Means of Porous Carbon Electrodes", J. Electrochem. Soc., vol. 118, pp. 510-517, 1971.
- [3] Y. Oren, "Capacitive deionization (CDI) for desalination and water treatment - past, present and future (a review)", Desalination, vol. 228, pp. 10-29, 2008.
- [4] L.Zubieta and R. Bonert, "Characterization of double-layer supercapacitors for power electronics applications", IEEE Trans. on Industry Applications, vol. 36, pp. 199-205, 2000.
- [5] P. Barrade, "Energy storage and applications with supercapacitors"
- [6] Z. Li, C. Jie, "An impedance-based approach to predict the state-ofcharge for carbon-based supercapacitors", Microelectronic Engineering, vol. 85, pp. 1549-1554, 2008.
- [7] O.H. LeBlanc, *Mathematics of Ultracapacitors*, GE Global Research, Technical Report, June 1993.
- [8] F.Belhachemi, S. Rael and B. Davat, "A physical based model of

power electric double-layer supercapacitors", Industrial Application Conference, vol. 5, pp. 3069-3076, 2000.

- [9] Conway, B. E. (1999). *Electrochemical Supercapacitors : Scientific Fundamentals and echnological Applications*. New York, Kluwer-Plenum.
- [10] Burke, A. (2000). "Ultracapacitors: why, how, and where is the technology." *Journal of Power Sources* 91(1): 37-50.
- [11] Kotz, R. and M. Carlen (2000). "Principles and applications of electrochemical capacitors." *ElectrochimicaActa* 45(15-16): 2483-2498.
- [12] R.Gallay, A. Schnewly and V. Hermann, "High performance double layer capacitors for power electronics applications", PCIM2001, Power Quality, Nurnberg, Germany